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Abstract. Using previous works on the second geometrization, we can now submit a method to use xTAN formalisme in order to study theoretically networks (xTAN means extended tensorial analysis of networks). A first application consists in the identification of risk in electromagnetic compatibility. But on this example, many others can be developed in various jobs including multiphysic ones. We detail the methodology through each of its steps: network graphs, equations, jacobian matrix and Kron's metric, linked parametrized surface, metric and sources, susceptibility spectrum, excitation spectrum and finally analysis.

1. Introduction

First tests to use geometry in network analysis was stopped. Due to formulation too much far from the available geometry theory, the Kron's initial equations do not give solution for this objective. Seconde geometrization encloses Kron's equations in standard differential geometry. As a consequence, it becomes possible to study networks under a general topological approach. We present here how to reach this kind of system of equations. After what we can submit a methodology that can be applied for all kind of problems. Finally we give some first simple examples in order to show how the technique can be used to analyse theoretically networks. In conclusion we speak of future works.

2. Basic mechanism to go from Kron's relations to xTAN

We won't develop here the Kron's method. Many publications are today available, including its specialization for EMC (MKME method: modified Kron's method for EMC which enclosed generalized interactions as chords). We suppose known the formalism. It leads, once the problem defined using graphs, to a group of equations giving the system behaviors.

2.1. Function ψ.

This system has the form:

$$\psi = \left\{ \psi_k \left(x^1, x^2, \ldots, x^n \right) = e_k \right\} \tag{2.1}$$

x^u are generalized variables in multiphysics, like currents, locations, speeds, temperatures, etc. e_k are sources of energy that can be included in the ψ_k functions. We prefer to separate them and keep them outside. Next operations justify this choice.

These equations are established from graphs. When various physics are involved, each kind of graph is usually associated with variables notations like i_n for electrical currents, T_i for temperatures, etc. All are grouped in an unique variable vector x^q leading to system (2.1).

2.2. **Parametrized surface.** We can see system (2.1) as a parametrized surface. A jacobian matrix can be construct based on:

$$J = [J_{ku}] = \frac{\partial \psi_k}{\partial x^u} \tag{2.2}$$

The jacobian matrix can be seen as a covector of the basic vector associated with the plane TpS locally tangent to the surface: $J = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \dots & \mathbf{b}_n \end{bmatrix}$ with $\mathbf{b}_u = (\partial_u \psi_k)$. $u \in \mathbb{N} = \{1, 2 \dots, N\}$ where N is the number of equations in ψ_k.

Something particular appears on component of time derivation: $\partial_t x^u$. As $[\partial_t, \partial_x] = 0$, it leads to $\partial_x \partial_t x = 0$ because $\partial_x \partial_t x = \partial_t \partial_x x = \partial_t 1 = 0$. So, all time derivatives coming from inductances doesn't give components to J[1].

2.3. **Writing ψ with J.** In order to obtain equations of ψ using J, we must add at least derivative components to something like $J_{ku} x^u$. More, J can include components that doesn't appear directly in ψ. If ψ_k can be writen:

$$R_{\mu\nu} x^\nu + L_{\mu\nu} x^\nu = e_\mu \tag{2.3}$$

and if $J_{\mu\nu} = R_{\mu\nu} + \zeta_{\mu\nu}$, we can obtain ψ through:

$$J_{\mu\nu} x^\nu - \zeta_{\mu\nu} x^\nu + L_{\mu\nu} x^\nu = e_\mu \tag{2.4}$$

and finally:

$$J_{\mu\nu} x^\nu = e_\mu - (L_{\mu\nu} - \zeta_{\mu\nu})\, x^\nu \tag{2.5}$$

2.4. **G coming from J.** By definition $G_{\mu\nu} = \langle \mathbf{b}_\mu, \mathbf{b}_\nu \rangle$. With (2.2), this leads to:

$$G = J^T J \tag{2.6}$$

This is the fundamental result giving the key to write ψ with G. If $J^T = \Upsilon$, (2.5) becomes:

$$G_{\sigma\nu} x^\nu = \Upsilon_{\sigma\mu} e_\mu - \Upsilon_{\sigma\mu} (L_{\mu\nu} - \zeta_{\mu\nu})\, x^\nu \tag{2.7}$$

Writing: $E_\sigma = \Upsilon_{\sigma\mu} e_\mu$ and $S_{\sigma\nu} = \Upsilon_{\sigma\mu} (L_{\mu\nu} - \zeta_{\mu\nu})$ we obtain:

$$G_{\sigma\nu} x^\nu = E_\sigma - S_{\sigma\nu} x^\nu = T_\sigma \tag{2.8}$$

$L_{\mu\nu}$ is the metric in the Kron's formalism and G the one in the last evolution of xTAN formalism. This last one is always symetric and in compliance with riemanian's concept. To study theoretically the network means to study G and its variation depending on time parameter and x^ν values.

[1] Note that it's the same if we write the system using Laplace's operator. If k is a constant, $pk = 0$ by definition, while $p^{-1} k \neq 0$: integration operations are included in J.

3. Manifolds associated with ψ and S meaning

Each ψ_k can be seen as a manifold (or sub-manifold) if associated with domains giving the limit values for x^ν. If G is fixed whatever x^ν values, it means that the abstract space of variables (including currents) is flat and to study its frontiers, it's only necessary to study the extreme values. If the abstract space of variables is curvilinear, the situation becomes more complex.

3.1. **Flat space.** If the abstract space of variables x^ν is flat, the tangent plane TpS stills the same whatever x^ν values. So if we look to some vector projection on TpS given by: $\mathbf{a} = a^k \mathbf{b}_k$. As there is a single plane for all the variables space, it means that we can consider it as an image of the space. The scalar product

$$\langle \mathbf{a}, \mathbf{b}_q \rangle = a^k G_{kq}$$

is the source covector T_q. If we look how change $G_{kq}a^q$ depending on x^q, it will give a constant value as TpS stills the same. Writing: $\theta_k = G_{kq}a^q$, we study here: $\partial_{x^q}\theta_k$.

Our problem in system reliability is to detect if the system states x^ν can reach values where the system can break down (or is simply be disturbed in electromagnetic compatibility). In other word the problem is to know if T_σ can be greather than $G_{\sigma\nu}x^\nu$ for the maxima values of x^ν and values coming from the environment for E_σ? Major problem comes from the fact that to external excitation, the system has its own inertia $L_{\sigma\nu}$ for which amplitudes depends on time variation. To evaluate the domains covered by the system functions, a solution can be to consider particular waveforms: the most severe that the system can accept. These are both fast rise time waveform and long time durations to combine high dynamic and high power signals.

Another method can be to make a statistic on all the signals used by the system. At each time step, the mean value can be assign to each variable x^ν or the mean value plus the deviation one.

Finally a third technique previously submit could be to give each x^ν a value taken from a set of possible ones recorded in all the functional life of the system.

From the output of these three approaches, we know x^ν and its maxima, also its time derivatives. Adding knwon values of E_σ gives the risk through the comparison of $G_{\sigma\nu}x^\nu$ and T_σ.

3.2. **Curved space.** A curved space is characterized by the fact that θ_k is not constant. Computing $\partial_{x^k}G_{kq}a^q$ gives two terms: $(\partial_{x^k}G_{kq})\,a^q + G_{kq}\,(\partial_{x^k}a^q)$. The metric variations are defined using first: $\mathbf{b}_{uv} = \partial_{x^u}\mathbf{b}_v$. For example, if $\mathbf{b}_1 = \left(2x^1, 0, 0\right)$ we obtain $\mathbf{b}_{11} = (2, 0, 0)$, with $G_{11} = 4(x^1)^2$. Once define the Christoffel's coefficients given by: $\Gamma_{ij,k} = \mathbf{b}_{ij} \cdot \mathbf{b}_k$. In our example, $\Gamma_{11,1} = 4x^1$. It leads to:

$$\frac{\partial}{\partial x^1} G_{11} = 2\Gamma_{11,1} \tag{3.1}$$

which can be generalized. This allows to compute θ_k and to study the system variation for various domains of x^k.

4. System strategy and trajectory through self adaptation

The system is parametrized by the time and eventually other parameters enclosed in ψ_k. But in real systems, functions can change also under non linear behaviors, self adaptability of complex systems, etc. It means that G can change in the system history. When L doesn't change, the risk can be raised up by the same comparison as previously. To apply changes on G we use what we call "gamma matrices". For example on a transformer, the metric seems like:

$$G = \begin{bmatrix} (R_1)^2 & 0 \\ 0 & (R_2)^2 \end{bmatrix}$$

Now for various reasons, R_1 can evolve to R_3 or R_4 with probability p_1 and p_2. Now calling:

$$G_1 = \begin{bmatrix} (R_3)^2 & 0 \\ 0 & (R_2)^2 \end{bmatrix} \quad G_2 = \begin{bmatrix} (R_4)^2 & 0 \\ 0 & (R_2)^2 \end{bmatrix}$$

The γ matrix of components $(\gamma, \tilde{\gamma})$ is:

$$\gamma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & G_1 G^{-1} & 0 \\ 0 & 0 & G_2 G^{-1} \end{bmatrix} \quad \tilde{\gamma} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & p_1 & 0 \\ 0 & 0 & p_2 \end{bmatrix}$$

γ is applied on what we call a tenfold: a list of objects that define the system. It encloses all matrices used by Kron's formalism, plus those added by the second geometrization (i.e. G, T) and an information vector $I = (1, 0, 0)$ memorying the probabilities of existing for each system state. If $\breve{u}$ is the system under study, we apply $\gamma \cdot \breve{u}$ to describe its evolution. Note that in our example, the γ matrix given acts only on G in $\breve{u}$. In general it can act on all the elements enclosed in $\breve{u}$. This technique allow to make the manifolds associated withe the system equations to change, to be deformed during time.

5. Methodology

The methodology follows next steps:

(1) to draw the graph and to choose a configuration space associating the primary variables (i, T, ...) to x^ν;
(2) to calculate J, L in order to find ψ;
(3) to make appearing G, S, T;
(4) to analyze the abstract space topology and properties.

6. Future works

Future works will consist in studying the various manifolds obtained under each kind of system and associated networks. Understanding their geometry, we may find global approach to demonstrate in which cases they are stable or unstable, when the risk is raised, etc. About the evoluation aspect, the work consists in understanding how the manifolds change depending on the human factors, and how the risk can appear after some steps of evolution. Global idea and approach is to go further in geometrization compare to previous works.